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# Smallness of the cosmological constant and the multiple point principle

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**Abstract.** In this talk we argue that the breakdown of global symmetries in no–scale supergravity (SUGRA), which ensures the vanishing of the vacuum energy density near the physical vacuum, leads to a natural realisation of the multiple point principle (MPP). In the MPP inspired SUGRA models the cosmological constant is naturally tiny.

## 1. No-scale supergravity and the multiple point principle

In $(N = 1)$ supergravity (SUGRA) models the scalar potential is specified in terms of the Kähler function

$$G(\phi_M, \phi_M^*) = K(\phi_M, \phi_M^*) + \ln |W(\phi_M)|^2 \,, \tag{1}$$

which is a combination of two functions: Kähler potential $K(\phi_M, \phi_M^*)$ and superpotential $W(\phi_M)$. Here we use standard supergravity mass units: $\frac{M_{Pl}}{\sqrt{8\pi}} = 1$. In order to break supersymmetry in $(N = 1)$ SUGRA models, a hidden sector is introduced. It is assumed that the superfields of the hidden sector $(z_i)$ interact with the observable ones only by means of gravity. At the minimum of the scalar potential, hidden sector fields acquire vacuum expectation values breaking local SUSY and generating a non–zero gravitino mass $m_{3/2} = < e^{G/2} >$.

In general the vacuum energy density in SUGRA models is huge and negative $\Lambda \sim -m_{3/2}^2 M_{Pl}^2$. The situation changes dramatically in no-scale supergravity where the invariance of the Lagrangian under imaginary translations and dilatations results in the vanishing of the vacuum energy density. Unfortunately these global symmetries also protect supersymmetry which has to be broken in any phenomenologically acceptable theory.

It was argued that the breakdown of dilatation invariance does not necessarily result in a non–zero vacuum energy density [1]–[2]. This happens if the dilatation invariance is broken in the superpotential of the hidden sector only. The hidden sector of the simplest SUGRA model of this type involves two singlet superfields, $T$ and $z$, that transform differently under dilatations

$$T \to \alpha^2 T \,, \qquad z \to \alpha z \tag{2}$$

The invariance under the global symmetry transformations (2) constrains the Kähler potential and superpotential of the hidden sector. In the considered SUGRA model they can be written in the following form [1]:

$$\hat{K} = -3 \ln \left[ T + \overline{T} - |z|^2 \right] \,, \qquad \hat{W}(z) = \kappa \left( z^3 + \mu_0 z^2 + \sum_{n=4}^{\infty} c_n z^n \right). \tag{3}$$

The bilinear mass term for the superfield $z$ and the higher order terms $c_n z^n$ in the superpotential $\hat{W}(z)$ spoil the dilatation invariance. However the SUGRA scalar potential of the hidden sector remains positive definite in the considered model

$$V(T, z) = \frac{1}{3(T + \overline{T} - |z|^2)^2} \left| \frac{\partial \hat{W}(z)}{\partial z} \right|^2 , \tag{4}$$

so that the vacuum energy density vanishes near its global minima. In the simplest case when $c_n = 0$, the scalar potential (4) has two extremum points at $z = 0$ and $z = -\frac{2\mu_0}{3}$. In the first vacuum where $z = -\frac{2\mu_0}{3}$, local supersymmetry is broken and the gravitino gains a non–zero mass. In the second minimum, the vacuum expectation value of the superfield $z$ and the gravitino mass vanish. If the high order terms $c_n z^n$ are present in Eq. (3), the scalar potential of the hidden sector may have many degenerate vacua with broken and unbroken supersymmetry in which the vacuum energy density vanishes.

Thus the considered breakdown of dilatation invariance leads to a natural realisation of the multiple point principle (MPP) assumption. The MPP postulates the existence of the maximal number of phases with the same energy density which are allowed by a given theory [3]. Successful application of the MPP to $(N = 1)$ supergravity requires us to assume the existence of a vacuum in which the low–energy limit of the considered theory is described by a pure supersymmetric model in flat Minkowski space [4]. According to the MPP this vacuum and the physical one in which we live must be degenerate. Such a second vacuum is realised only if the SUGRA scalar potential has a minimum where $m_{3/2} = 0$ which normally requires an extra fine-tuning [4]. In the SUGRA model considered above the MPP conditions are fulfilled automatically without any extra fine-tuning.

## 2. Cosmological constant in the MPP inspired SUGRA models

Since the vacuum energy density of supersymmetric states in flat Minkowski space is just zero and all vacua in the MPP inspired SUGRA models are degenerate, the cosmological constant problem is solved to first approximation by assumption in these models. However the value of the cosmological constant may differ from zero in the considered models. This occurs if non–perturbative effects in the observable sector give rise to the breakdown of supersymmetry in the second vacuum (phase). The MPP philosophy then requires that the physical phase in which local supersymmetry is broken in the hidden sector has the same energy density as a second phase where non–perturbative supersymmetry breakdown takes place in the observable sector.

If supersymmetry breaking takes place in the second vacuum, it is caused by the strong interactions. When the gauge couplings at high energies are identical in both vacua and $M_S$ is the SUSY breaking scale in the physical vacuum the scale $\Lambda_{SQCD}$, where the QCD interactions become strong in the second vacuum, is given by

$$\Lambda_{SQCD} = M_S \exp\left[ \frac{2\pi}{b_3 \alpha_3^{(2)}(M_S)} \right] , \qquad \frac{1}{\alpha_3^{(2)}(M_S)} = \frac{1}{\alpha_3^{(1)}(M_Z)} - \frac{\tilde{b}_3}{4\pi} \ln \frac{M_S^2}{M_Z^2} . \tag{5}$$

In Eq.(5) $\alpha_3^{(1)}$ and $\alpha_3^{(2)}$ are the values of the strong gauge couplings in the physical and second minima of the SUGRA scalar potential while $\tilde{b}_3 = -7$ and $b_3 = -3$ are the one–loop beta functions of the SM and MSSM. At the scale $\Lambda_{SQCD}$ the t–quark Yukawa coupling in the MSSM is of the same order of magnitude as the strong gauge coupling. The large Yukawa coupling of the top quark may result in the formation of a quark condensate that breaks supersymmetry inducing a non–zero positive value for the cosmological constant $\Lambda \simeq \Lambda_{SQCD}^4$.

In Fig. 1 the dependence of $\Lambda_{SQCD}$ on the SUSY breaking scale $M_S$ is examined. When the supersymmetry breaking scale in our vacuum is of the order of 1 TeV, we obtain $\Lambda_{SQCD} =$
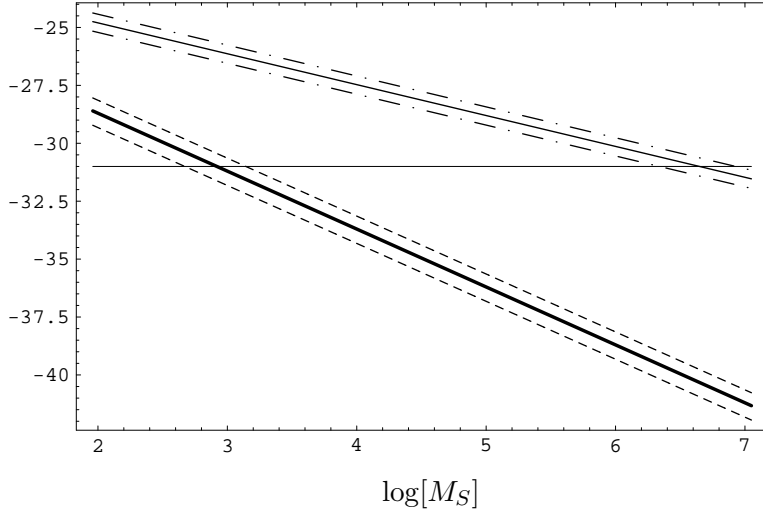
$\log[\Lambda_{SQCD}/M_{Pl}]$



**Figure 1.** The value of $\log[\Lambda_{SQCD}/M_{Pl}]$ versus $\log M_S$. The thin and thick solid lines correspond to the pure MSSM and the MSSM with an extra pair of $5 + \bar{5}$ multiplets. The dashed and dash–dotted lines represent the uncertainty in $\alpha_3(M_Z)$, i.e. $\alpha_3(M_Z) = 0.112 - 0.124$. The horizontal line corresponds to the observed value of $\Lambda^{1/4}$. The SUSY breaking scale $M_S$ is given in GeV.

$10^{-26}M_{Pl} \simeq 100$ eV [1]. This results in an enormous suppression of the total vacuum energy density ($\Lambda \simeq 10^{-104}M_{Pl}^4$) compared to say an electroweak scale contribution in our vacuum $v^4 \simeq 10^{-62}M_{Pl}$. From the rough estimate of the energy density it can be easily seen that the measured value of the cosmological constant is reproduced when $\Lambda_{SQCD} = 10^{-31}M_{Pl} \simeq 10^{-3}$ eV. The appropriate values of $\Lambda_{SQCD}$ can therefore only be obtained for $M_S = 10^3 - 10^4$ TeV [1]. However if the MSSM particle content is supplemented by an additional pair of $5 + \bar{5}$ multiplets the observed value of the cosmological constant can be reproduced even for $M_S \simeq 1$ TeV (see Fig. 1). In the physical vacuum these extra particles would gain masses around the supersymmetry breaking scale due to the presence of the bilinear term $[\eta(5 \cdot \bar{5}) + h.c.]$ in $K(\phi_M, \phi_M^*)$. Near the second minimum of the SUGRA scalar potential the new particles would be massless, since $m_{3/2} = 0$, and would reduce $\Lambda_{SQCD}$ via their contribution to the $\beta$ functions.

## 3. Conclusions
We have shown that the breakdown of global symmetries in no-scale supergravity can lead to a set of degenerate vacua with broken and unbroken local supersymmetry (first and second phases) so that the MPP conditions are satisfied without any extra fine-tuning. In the MPP inspired SUGRA models supersymmetry in the second phase may be broken dynamically in the observable sector inducing a very small positive energy density which can be assigned, by virtue of the MPP, to all other phases. In such a way we have suggested an explanation of why the observed value of the cosmological constant is positive and takes on the tiny value it has.

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